

*fast*NLLO

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- Motivation
- Concept
- The Product
- Status / Outlook

- Computations of higher-order pQCD predictions for hadronic-final state observables are time-consuming
- Often need repeated computations of the same cross section for different PDFs and/or $\alpha_s(M_z)$ values
- Examples for a specific analysis:
 - use various PDFs (CTEQ, MRST, Alekhin, Botje, H1, ZEUS, ...)
 - determine PDF uncertainties (PDF error sets)
 - use data set in fit of PDFs and/or α_s
- For some observables NLO predictions can be computed extremely fast (e.g.: DIS structure functions)
- ... but some are extremely slow: Drell-Yan and Jet Cross Sections

⇒ need new procedure for very fast repeated computations of NLO cross sections

- Can be used for **any** observable in hadron-induced processes (hadron-hadron / DIS / photoproduction)
- Although labeled “fastNLO” → can be used in any order ⇒ fastNⁿLO
- Our concept does not include the theoretical calculation itself (leave this to theorists) → it requires existing flexible computer code — here: NLOJET++ (Zoltan Nagy)
- During the first computation no time is saved
need full time of the original code: hours, days, weeks, months, ...
to achieve high statistical precision
- This concept involves one single approximation (see later)
But: precision of approximation can be quantified & arbitrarily improved
- Any further computation takes **one second** (independent of statistical precision)

⇒ **here:** example for inclusive jet production in hadron-hadron collisions

k-factor approximation:

- for a given PDF \rightarrow compute k-factor for each bin: $k = \sigma(\text{NLO})/\sigma(\text{LO})$
- “relatively fast”: compute LO cross section for arbitrary PDF
- multiply $\sigma(\text{LO})$ with k-factor \rightarrow get “NLO” prediction

problem:

- k-factor itself depends on the PDFs $\longrightarrow \longrightarrow \longrightarrow$
- higher for gluon induced subprocesses

reason:

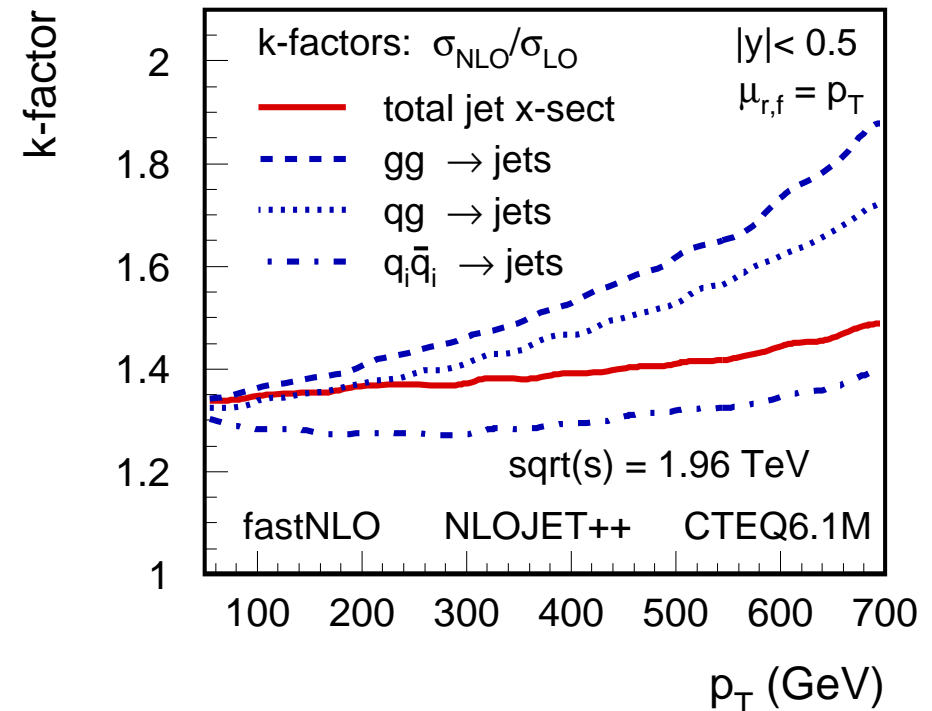
- different x-coverage in LO and NLO
- different k-factors for different subprocesses

limitations:

- even the LO computation is slow
- computing time depends on statistical precision



- as exact as you like
- much, much faster



General cross section formula for hadron-hadron collisions:

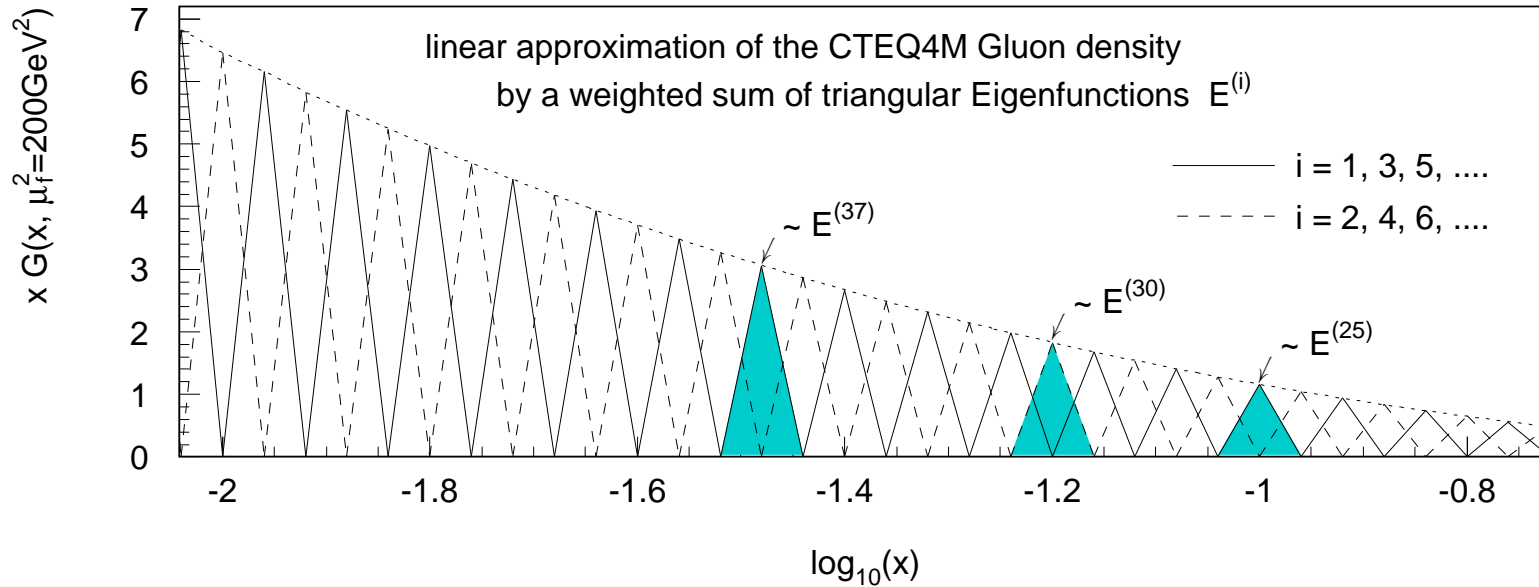
$$\sigma_{\text{hh}} = \sum_n \alpha_s^n(\mu_r) \sum_{\text{PDFflavors } i} \sum_{\text{PDFflavors } j} c_{i,j,n}(\mu_r, \mu_f) \otimes f_i(x_1, \mu_f) \otimes f_j(x_2, \mu_f).$$

- strong coupling constant α_s in order n
- perturbative coefficient $c_{i,j,n}$
- parton density functions (PDFs) of the hadrons $f_i(x)$, $f_j(x)$
- renormalization scale μ_r , factorization scale μ_f , momentum fraction x

Standard procedure:

- integration over whole phase space (x_1, x_2) (usually Monte-Carlo method)
- at each MC integration point:
 - computation of observable (e.g. run jet algorithm, determine p_T , $|y|$ bin)
 - compute perturbative coefficient
 - get α_s and PDFs values
 - ⇒ add contribution to bin

goal: try to separate the PDFs from the integral



- introduce a set of discrete x -values labeled $x^{(i)}$ ($i = 0, 1, 2, \dots, n$)
- with $x^{(n)} < x^{(n-1)} < x^{(n-2)} < \dots < x^{(0)} = 1$
- around each $x^{(i)}$, define an eigenfunction $E^{(i)}(x)$
- with $E^{(i)}(x^{(i)}) = 1, E^{(i)}(x^{(j)}) = 0$ for $i \neq j$ and $\sum_i E^{(i)}(x) = 1$ for all x
- express a single PDF $f(x)$ by a linear combination of eigenfunctions $E^{(i)}(x)$ with coefficients given by the PDF values $f(x^{(i)})$ at the discrete points $x^{(i)}$

$$f(x) = \sum_i f(x^{(i)}) E^{(i)}(x)$$

processes with two hadrons – need Eigenfunctions in 2d-space (x_1, x_2)

➤ define $E^{(i,j)}(x_1, x_2) \equiv E^{(i)}(x_1)E^{(j)}(x_2)$

➤ product of two PDFs $f(x_1, x_2) \equiv f_1(x_1) f_2(x_2)$ is given by

$$f(x_1, x_2) = \sum_{i,j} f(x_1^{(i)}, x_2^{(j)}) E^{(i,j)}(x_1, x_2)$$

note: this is an **approximation!!**

choice of triangular Eigenfunctions \implies linear interpolation of PDFs between adjacent $x^{(i)}$

this is the **only** approximation in fastNLO — precision can be arbitrarily improved!!

precision depends on:

- choice of set of $x^{(i)}$ — e.g. on $\log_{10}(1/x)$ or $\sqrt{\log_{10}(1/x)}$ (needs clever choice)
- number of x-bins (brute force) \longrightarrow memory $\propto n^2$

\implies **goal:** precision of 0.3% for all bins

now: don't want to deal with 13×13 PDFs!!

For hadron-hadron \rightarrow jets there are **seven** relevant partonic subprocesses:

$gg \rightarrow$ jets		\propto	$H_1(x_1, x_2)$
$qq \rightarrow$ jets	plus	$\bar{q}g \rightarrow$ jets	\propto $H_2(x_1, x_2)$
$gq \rightarrow$ jets	plus	$g\bar{q} \rightarrow$ jets	\propto $H_3(x_1, x_2)$
$q_i q_j \rightarrow$ jets	plus	$\bar{q}_i \bar{q}_j \rightarrow$ jets	\propto $H_4(x_1, x_2)$
$q_i q_i \rightarrow$ jets	plus	$\bar{q}_i \bar{q}_i \rightarrow$ jets	\propto $H_5(x_1, x_2)$
$q_i \bar{q}_i \rightarrow$ jets	plus	$\bar{q}_i q_i \rightarrow$ jets	\propto $H_6(x_1, x_2)$
$q_i \bar{q}_j \rightarrow$ jets	plus	$\bar{q}_i q_j \rightarrow$ jets	\propto $H_7(x_1, x_2)$

The H_i are linear combinations of PDFs
 \rightarrow reduced from 13×13 to seven!!

detail:

for hadron - anti-hadron collisions:

PDFs of the anti-hadron are expressed by the PDFs of the hadron (quarks \leftrightarrow anti-quarks)

here: swap $H_4 \leftrightarrow H_7$ and $H_5 \leftrightarrow H_6$

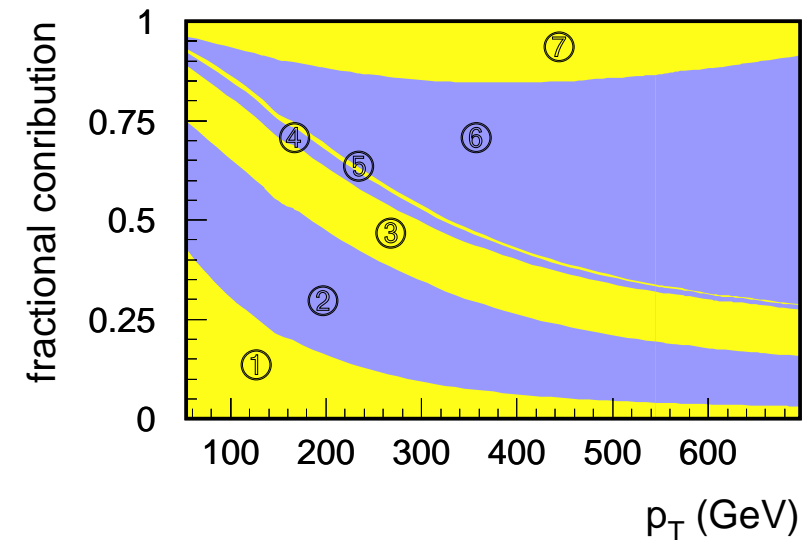
partonic subprocesses for $p\bar{p} \rightarrow$ jets

sqrt(s) = 1.96 TeV

$|y| < 0.5$

fastNLO
 NLOJET++ / CTEQ6.1M

- ⑦ $q_i \bar{q}_i \rightarrow$ jets
- ⑥ $q_i \bar{q}_j \rightarrow$ jets
- ⑤ $q_i q_i \rightarrow$ jets
- ④ $q_i q_j \rightarrow$ jets
- ③ $gq \rightarrow$ jets ($x_g > x_q$)
- ② $gq \rightarrow$ jets ($x_g < x_q$)
- ① $gg \rightarrow$ jets



$$G(x, \mu_f) = g(x, \mu_f)$$

$$Q(x, \mu_f) = \sum_i q_i(x, \mu_f)$$

$$\bar{Q}(x, \mu_f) = \sum_i \bar{q}_i(x, \mu_f)$$

$$S(x_1, x_2, \mu_f) = \sum_i (q_i(x_1, \mu_f) q_i(x_2, \mu_f) + \bar{q}_i(x_1, \mu_f) \bar{q}_i(x_2, \mu_f))$$

$$A(x_1, x_2, \mu_f) = \sum_i (q_i(x_1, \mu_f) \bar{q}_i(x_2, \mu_f) + \bar{q}_i(x_1, \mu_f) q_i(x_2, \mu_f))$$

- $q_i(x)$ ($\bar{q}_i(x)$) — quark (anti-quark) density of flavor i
- $i = 1, \dots, n_f$ — No. of flavors
- $G(x)$ — gluon density

$$\begin{aligned}
 H_1(\mathbf{x}_1, \mathbf{x}_2) &= G(\mathbf{x}_1) G(\mathbf{x}_2) , \\
 H_2(\mathbf{x}_1, \mathbf{x}_2) &= (Q(\mathbf{x}_1) + \bar{Q}(\mathbf{x}_1)) G(\mathbf{x}_2) , \\
 H_3(\mathbf{x}_1, \mathbf{x}_2) &= G(\mathbf{x}_1) (Q(\mathbf{x}_2) + \bar{Q}(\mathbf{x}_2)) , \\
 H_4(\mathbf{x}_1, \mathbf{x}_2) &= Q(\mathbf{x}_1)Q(\mathbf{x}_2) + \bar{Q}(\mathbf{x}_1)\bar{Q}(\mathbf{x}_2) - S(\mathbf{x}_1, \mathbf{x}_2) , \\
 H_5(\mathbf{x}_1, \mathbf{x}_2) &= S(\mathbf{x}_1, \mathbf{x}_2) , \\
 H_6(\mathbf{x}_1, \mathbf{x}_2) &= A(\mathbf{x}_1, \mathbf{x}_2) , \\
 H_7(\mathbf{x}_1, \mathbf{x}_2) &= Q(\mathbf{x}_1)\bar{Q}(\mathbf{x}_2) + \bar{Q}(\mathbf{x}_1)Q(\mathbf{x}_2) - A(\mathbf{x}_1, \mathbf{x}_2) .
 \end{aligned}$$

These are the seven combinations of PDFs, corresponding to the seven subprocesses

symmetries:

$$H_n(\mathbf{x}_1, \mathbf{x}_2) = H_n(\mathbf{x}_2, \mathbf{x}_1) \text{ for } n = 1, 4, 5, 6, 7 \quad \text{and} \quad H_2(\mathbf{x}_1, \mathbf{x}_2) = H_3(\mathbf{x}_2, \mathbf{x}_1)$$

$$H_k(\mathbf{x}_1, \mathbf{x}_2) = \sum_{(i,j)} H_k(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}) E^{(i,j)}(\mathbf{x}_1, \mathbf{x}_2)$$

where $H_k(\mathbf{x}^{(i)}, \mathbf{x}^{(j)})$ is a number \leftrightarrow PDF information

With these definitions of the seven H_i the cross section reads:

$$\sigma_{\text{hh}} = \sum_n \alpha_s^n(\mu_r) \sum_{k=1}^7 c_{k,n}(\mu_r, \mu_f) \otimes H_k(\mathbf{x}_1, \mathbf{x}_2, \mu_f)$$

Now: express H_k by linear combinations of the $E^{(i,j)}(\mathbf{x}_1, \mathbf{x}_2)$

$$\sigma_{\text{hh}} = \sum_n \alpha_s^n(\mu_r) \sum_{k=1}^7 c_{k,n}(\mu_r, \mu_f) \otimes \left(\sum_{i,j} H_k(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}) \cdot E^{(i,j)}(\mathbf{x}_1, \mathbf{x}_2) \right)$$

or, better:

$$\sigma_{\text{hh}} = \sum_n \alpha_s^n(\mu_r) \sum_{k=1}^7 \sum_{i,j} H_k(\mathbf{x}_1^{(i)}, \mathbf{x}_2^{(j)}) \left(c_{k,n}(\mu_r, \mu_f) \otimes E^{(i,j)}(\mathbf{x}_1, \mathbf{x}_2) \right)$$

important: integral is independent of PDFs!

the numbers $H_k(\mathbf{x}^{(i)}, \mathbf{x}^{(j)})$ contain all information on the PDFs

⇒ exactly what we wanted!!

define:

$$\tilde{\sigma}_{k,n}^{(i,j)} \equiv c_{k,n}(\mu_r, \mu_f) \otimes E^{(i,j)}(\mathbf{x}_1, \mathbf{x}_2)$$

⇒ the $\tilde{\sigma}_{k,n}^{(i,j)}$ contain all information on the observable

(the perturbative coefficients, the jet definition, and the phase space restrictions).

but: $\tilde{\sigma}_{k,n}^{(i,j)}$ is independent of the PDFs and α_s – needs to be computed only once!

The cross section is then given by the simple product (→ **Master Formula!**)

$$\sigma_{\text{hh}} = \sum_{i,j,k,n} \alpha_s^n(\mu_r) H_k(\mathbf{x}_1^{(i)}, \mathbf{x}_2^{(j)}) \tilde{\sigma}_{k,n}^{(i,j)}$$

can be reevaluated **very** quickly for different PDFs and α_s values,

as e.g. required in the determination of PDF uncertainties or in global fits of PDFs

to implement a new observable in fastNLO:

- find theorist to provide flexible computer code
- identify elementary subprocesses & relevant PDF linear combinations
- define analysis bins (e.g. p_T , $|\mathbf{y}|$)
- define Eigenfunctions $E(\mathbf{x})$, $E(\mathbf{x}_1, \mathbf{x}_2)$ (e.g. triangular) & the set of $\mathbf{x}^{(i)}$
- to optimize x-range: find lower x-limit ($\mathbf{x}_{\text{limit}} < \mathbf{x} < 1$) (for each analysis bin)

example: DØ Run I measurement of Incl. Jet Cross Section, Phys. Rev. Lett.86, 1707 (2001)

- 90 analysis bins in (E_T, η)
- 2 orders of $\alpha_s(p_T)$ (LO & NLO)
- 7 partonic subprocesses
- No. of x-intervals for each bin: 50 (100?) ← (study precision of PDF approximation)
 $\Rightarrow (n^2 + n)/2 = 1275$ (5050?) Eigenfunctions $E^{(i,j)}(\mathbf{x}_1, \mathbf{x}_2)$
- compute 1.6M (6.4M?) variables $\tilde{\sigma}_{k,n}^{(i,j)}$ (times three, if scale variations are included)
 \Rightarrow stored in huge table!!!

compute VERY long to achieve very high precision — (after all: needs to be done only once!)

Everything will be downloadable from the **fastNLO** Webpage

Package for a single observable includes:

- Tables of $\tilde{\sigma}_{k,n}^{(i,j)}$ in different orders - for different scales
- Stand-Alone Code to:
 - ✕ read tables
 - ✕ loop over PDFs (LHAPDFlib interface - or custom user interface for global fitters)
 - ✕ output cross section numbers as: array, ASCII, ROOT/HBOOK histograms
- Examples

Code computes NLO Predictions for a whole set of data points in the order of seconds (depends on speed of PDF interface)

Can easily be included into user-specific analysis framework

Status:

- concept for **fastNLO** is fully developed
- implementation of code for hadron-hadron jet cross section finished
- currently: studying precision / x-binning / “tweaking”

Outlook:

(start with inclusive jet production)

- first: provide tables and user code for published Run I results from CDF and DØ at 630 GeV and 1800 GeV — in analysis specific bins
(→ data can easily be included in all PDF fits)
- next: provide tables and user code for Run II and LHC energies – flexible in p_T, y
 - need to know: reasonable (p_T, y) binning for LHC (?)
 - for different jet algorithms – which jet algorithm(s) will be used at the LHC (?)
- later: extend to dijet production / Drell-Yan@NNLO / . . . ???

⇒ **first results by summer**